Multi-Valued and Probabilistic Argumentation Frameworks

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Abstract. In this paper we further progress the analysis of the recently introduced multi-valued argumentation frameworks ($MVAFs$). $MVAFs$ are an extension of Dung's abstract argumentation, where arguments have a degree of truth associated with them. Here we describe a list of properties of $MVAFs$ considering the major multi-valued logics such as those proposed by Gödel, Zadeh and Łukasiewicz. We then propose a computational framework that joins multi-valued and probabilistic argumentation frameworks to handle situations where arguments affected by vagueness and/or probabilities coexist. The findings are a contribution to the field of non-monotonic approximate reasoning and they also represent a well-grounded proposal towards the introduction of gradualism in argumentation systems.

Keywords. Multi-valued logics, Probabilistic Reasoning, Abstract Argumentation

Introduction

An abstract argumentation framework is a direct graph where nodes represent arguments and arrows represent the attack relation. These frameworks were introduced to analyse defeasible arguments and study conflict resolution strategies among them.

In Dung's original work, arguments are either fully asserted or not asserted at all, and as a consequence abstract argumentation is often too strict and coarse to support a decision-making process. Recent approaches [3, 4] have tried to marry abstract argumentation and probability calculus, defining probabilistic argumentation frameworks ($PAFs$). In a PAF each argument has a probability associated with it, measuring (in the mainstream interpretation) the likelihood of the premise of the argument to be true. Following a similar conceptual framework, in [12] we investigated how abstract argumentation and multi-valued logics could be married to handle gradual arguments. This is the case for arguments structured as inference rules containing fuzzy terms, such as *"if the tomato is rotten, do not it eat"*.

Both probabilistic and multi-valued frameworks induce multiple scenarios over the starting argumentation framework. In the case of $PAFs$, the probabilistic nature of the premises of the arguments identifies multiple mutually exclusive scenarios, each of them obtained by assuming some arguments to hold and some others not to hold. In the case of a $MVAF$, premises of arguments are *gradual* and therefore, at the same time, they are satisfied to a certain degree and not satisfied to another degree. Again, multiple scenarios are therefore possible, this time coexisting and not mutually exclusive. Both the computational analyses of $PAFs$ and $MVAFs$ are centred on the analysis of the subgraphs of the original argumentation graph. However, dissimilarities remain due to the different properties of probability calculus and multi-valued logics. In this paper we describe a set of properties of $MVAFs$ considering the major multi-valued logics such as proposed by Gödel, Zadeh and Łukasiewicz. We compare the findings with analogous properties identified for $PAFs$. We then propose a computational framework that joins multi-valued and probabilistic argumentation frameworks to handle situations where vagueness and probabilistic uncertainty coexist. The paper is organized as follows: the next section presents the required background of abstract argumentation and multi-valued logic; section 2 describes $PAFs$ and $MVAFs$ analyzing their differences and properties; section 3 describes our approach to join the two frameworks, and this is followed by a related works section and conclusions.

1 Background

1.1. Background Definitions

Definition 1. An argumentation framework AF is a pair (Ar, R) , where Ar is a non-empty finite set whose elements are called arguments and $R \subseteq Ar \times Ar$ a binary relation, called the attack relation. If $(a, b) \in R$ we say that a attacks b. Two arguments a, b are **rebuttals** *iff* $(a, b) \in R \wedge (b, a) \in R$.

Definition 2. *(conflict-free). $Args$ is conflict-free iff $\nexists a, b \in Args \mid a\ R\ b$.*

Definition 4. *(admissible set). $Args$ defends an argument $a \subseteq Ar$ iff $\forall b \in Ar$ such that $(b, a) \in R$, $\exists\, c \in Args$ such that $(c, b) \in R$. The set of arguments defended by $Args$ is denoted $F(Args)$. A set $Args$ is admissible if $Args \subseteq F(Args)$ and it is complete if $Args = F(Args)$*

An abstract argumentation semantics identifies a set of arguments that can survive the conflicts encoded by the attack relation R. We follow the labelling approach of [5], where a semantics assigns to each argument a label *in*, *out* or *undec*, meaning that the argument is considered consistently acceptable, non-acceptable or undecided

Definition 3. *(labelling). Let $AF = (Ar, R)$. A labelling is a total function $L : Ar \rightarrow \{in, out, undec\}$. We write in(L) for $\{a \in Ar | L(a) = in\}$, out(L) for $\{a \in Ar | L(a) = out\}$, and undec(L) for $\{a \in Ar | L(a) = undec\}$.*

Definition 4. *(complete labelling, from Definition 5 in [5]). Let (Ar, R) be an argumentation framework. A complete labelling is a labelling that for every $a \in Ar$ holds that:1. if a is labeled in then all attackers of a are labeled out; 2. if all attackers of a are labeled out then a is labeled in; 3. if a is labeled out then a has an attacker labeled in; 4. if a has an attacker labeled in then a is labeled out*

Theorem 1. [5] *Let L be a labelling of argumentation framework (Ar, R). It holds that L is a complete labelling iff for each argument $a \in Ar$ it holds that: 1. if a is labeled in then all its attackers are labeled out; 2. if a is labeled out then it has at least one attacker that is labeled in; 3. if a is labeled $undec$ then it has at least one attacker that is labeled $undec$ and it does not have an attacker that is labeled in.*

Theorem 2. *(from th. 6 and 7 in* [5]*) Given $AF = (Ar, R)$, L is the grounded labelling iff L is a complete labelling where undec(L) is maximal (w.r.t. set inclusion) among all complete labellings of AF.*

In figure 1 two argumentation graphs are depicted. Grounded semantics assigns the status of *undec* to all the arguments of the argumentation framework on the left, since it represents the complete labelling with the maximal set, while in the framework on the right, according to theorem 1, there is only one complete labelling (thus grounded), where argument a is *in* (no attackers), b is *out* and c is *in*. Note how a reinstates c.

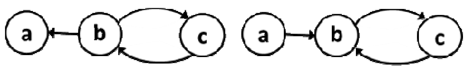


Figure. 1. Two Argumentation Graphs (1) and (2)

1.2. Subgraph Notation and Labelling of Subgraphs of an AF

Given an argumentation $AF = (Ar, R)$ with $|Ar| = n$, and the graph $\mathscr{g}$ identified by Ar and R, we consider the set $\mathcal{H}$ of all the subgraphs of $\mathscr{g}$. We focus on particular sets of subgraphs, i.e. elements of $2^{\mathcal{H}}$. Given $a \in Ar$, we define:

$$A = \{\mathscr{h} \in \mathcal{H} \mid a \text{ is a node of } \mathscr{h}\} \quad ; \quad \bar{A} = \{\mathscr{h} \in \mathcal{H} \mid a \text{ is not a node } of\ \mathscr{h}\}$$

A and $\bar{A}$ are respectively the set of subgraphs where argument a is present and the complementary set of subgraphs where a is not present. If $Ar = \{a_1, .., a_n\}$, a single subgraph $\mathscr{h}$ can be expressed by an intersection of n sets A_i or $\overline{A_i}$ $(i \leq n)$ depending on whether the i^{th} argument a_i is or is not contained in the set of nodes of $\mathscr{h}$. A set of subgraphs can be expressed by combining some of the sets $A_1, .., A_n, \overline{A_1}, .., \overline{A_n}$. with the connectives $\{\cup, \cap\}$. We write AB to denote $A \cap B$ and $A + B$ for $A \cup B$. For instance, in figure 1 left the single subgraph with only b and c present is denoted with $\bar{A}BC$, while the expression AB denotes a set of two subgraphs where arguments a and b are present and the status of c (not in the expression AB) is indifferent.

Given a subgraph $\mathscr{h} \in \mathcal{H}$, the labelling of $\mathscr{h}$ follows the rules of the chosen semantics. We therefore define a *subgraph labelling* $\mathcal{L}$ as a total function over the Cartesian product of arguments in Ar and subgraphs in $\mathcal{H}$, therefore $\mathcal{L}: Ar \times \mathcal{H} \rightarrow \{in, out, undec\}$. When labelling a subgraph, an argument a is automatically labelled *out* in all the subgraphs where a is not present (since it does not promote any claim) *or* when it is present but it is labelled out by the semantics, representing the effect on a of the other arguments (see [11] for a detailed justification).

In the case of grounded semantics there is only one labelling per subgraph $\mathscr{h}$, and this we call $\mathcal{L}(\mathscr{h})$ (we omit Ar). We call $in(\mathcal{L}(\mathscr{h}))$, $out(\mathcal{L}(\mathscr{h}))$, $undec(\mathcal{L}(\mathscr{h}))$ the sets of arguments labelled in, out, $undec$ in the labelling $\mathcal{L}(\mathscr{h})$. In order to study how an argument behaves across subgraphs in $\mathcal{H}$, we define these sets of subgraphs:

$$\forall a \in Ar\ (A_{IN} = \{\mathscr{h} \in \mathcal{H} : a \in in(\mathcal{L}(\mathscr{h}))\}, A_{OUT} = \{\mathscr{h} \in \mathcal{H} : a \in out(\mathcal{L}(\mathscr{h}))\},$$
$$A_U = \{\mathscr{h} \in \mathcal{H} : a \in undec(\mathcal{L}(\mathscr{h}))\})$$

i.e. the sets of subgraphs where a is labelled *in, out, undec.*

Example 1. In the graph of figure 1 left, there are 3 arguments and 2^3 subgraphs; argument a is labelled in in all the subgraphs where a is present and b is not present (and c becomes irrelevant), i.e. $A_{IN} = A\bar{B}$. It is $undec$ when all the arguments are present (the single subgraph $A_U = ABC$) while a is out when it is not present or when b is present and c is not present, i.e. $A_{OUT} = \bar{A} + AB\bar{C}$.

1.3. Multi-valued Logic

In the setting of multi-valued logic, a sentence is not true or false only, but may have a truth degree taken from an ordered scale, called truth space S, such as [0,1]. Multi-valued logic can model situations affected by vagueness, where a statement is often satisfied to a certain extent and the concepts discussed are graded. This is usual in natural language when words are modeled by fuzzy sets, such as *tall, young* or *fast*. In this context we identify a proposition with a fuzzy set and the degree of membership of a state of affairs to this fuzzy set evaluates the degree of fit between the proposition and the state of facts it refers to. This degree of fit is called *degree of truth* of a proposition ϕ. Semantically, a many-valued interpretation I maps each basic proposition ϕ, ψ into [0,1] and is then extended inductively as follows:

$$I(\phi \wedge \psi) = I(\phi) \otimes I(\psi) \quad ; \quad I(\phi \vee \psi) = I(\phi) \oplus I(\psi)$$
$$I(\phi \rightarrow \psi) = I(\phi) \rhd I(\psi) \quad ; \quad I(\bar{\phi}) = \ominus\, I(\psi)$$

where $\otimes$, $\oplus$, $\rhd$ and $\ominus$ are called triangular norms, triangular co-norms, implication functions, and negation functions respectively, which extend the classical Boolean conjunction, disjunction, implication, and negation to the many-valued case. These functions have all to satisfy the following properties: tautology, contradiction, commutativity, associativity and monotonicity, but not all of them satisfy excluded middle ($x \otimes \ominus x = 0$) or double negation ($\ominus\ominus x=x$). We distinguish two main logics: Łukasiewicz's and Gödel's logic; Zadeh's logic is a sublogic of Łukasiewicz's logic. Their operators are shown in table 1. For a good analysis see [10].

Table 1. Combination functions of various fuzzy logics

	Łukasiewicz's L.	Gödel's logic	Zadeh's logic
a⊗b	max (a+b−1,0)	min (a,b)	min (a,b)
a⊕b	min (a+b,1)	max (a,b)	max (a,b)
⊖a	1−a	$\begin{cases} 1 \text{ if } a = 0 \\ 0 \text{ otherwise} \end{cases}$	1−a

2 Gradualism and Probabilities in Abstract Argumentation

Let us presume our argumentation framework includes n arguments and that each argument is an inference rule between propositions of a language. If these propositions are affected by uncertainty or gradualism, we are not sure if the claim of each argument can be used in the argumentation process. If the proposition ϕ representing a claim is probabilistic, it can hold or not; if ϕ is gradual, it partially holds (and partially not). The consequence is that multiple scenarios of the same argumentation process have to be taken into account and each scenario is described by a subset of the original argumentation framework.

The case of probabilistic uncertainty has been recently analyzed in [4] and [3] and [11]. We use the following definition of PAF (from [11], modified from [4]):

Definition 5 (PAF). *A probabilistic argumentation framework PAF is a couple* (A, P) *where* $A = (Ar, R)$ *is an abstract argumentation framework and* $P: 2^{Ar} \rightarrow (0,1]$ *is a joint probability distribution over* Ar.

In a PAF arguments have a probability attached to them, indicating the likelihood of the argument to hold (based on the probability to which its premises are true, or are believed to be true). Since the premises are affected by probabilistic uncertainty, the premises are satisfied (and the claim follows) in a subset of situations with likelihood x, and they are not satisfied in the complementary set of situations (with likelihood $1 - x$). Given an argumentation graph with n arguments, there are 2^n possible situations, each of them identifying a subgraph of the original argumentation graph. Li [3] calls these situations *induced argumentation frameworks*. Each induced framework behaves as an abstract Dung-style framework and it has a probability attached to it, computed using the (joint) probability distribution P defined over the arguments. Given a semantics, the probability of an argument a being labelled *in* (or *out* or *undec*) is the sum of the probabilities of all the induced frameworks where the chosen semantics produces the required label for a. This computation is referred to in [4] as the *constellation approach.* We call $P_{A_{IN}}$ the probability of argument a being labeled *in*, while P_A is the probability of argument a being part of a subgraph, equal to the probability of the set of subgraph A.

In a multi-valued argumentation setting, arguments have a degree of truth in [0,1] attached to them, indicating extent to which their premises (and therefore claims) are compatible with a particular state of affairs. This is the case for arguments structured as inference rules containing fuzzy terms, such as *"if the tomato is rotten, do not eat it"*. The support and therefore the claim of the argument assumes different degrees of truth when applied to different tomatoes. A $MVAF$ is defined as follows:

Definition 6 $\boldsymbol{MVAF}$ *A multi-valued argumentation framework (MVAF) is a tuple* $((Ar, R), \mu)$ *where* (Ar, R) *is an abstract argumentation framework and* $\mu: Ar \rightarrow [0,1]$ *assigns a degree of truth to each argument in* Ar.

We write μ_A as a shortcut for $\mu(a)$. The degree to which an argument a is labelled in (or *out* or *undec*) is called $\mu_{A_{IN}}$ ($\mu_{A_{OUT}}, \mu_{A_U}$).

If a claim has a degree of truth μ attached to it, this means that the current *state of affairs* satisfies the claim to a certain degree μ but at the same time it also satisfies the negation of the claim with a degree quantified by the negation operator $\ominus$. These values do not refer to two distinct situations – as in the case of probabilistic uncertainty – but they represent degrees of truth attached to two co-existing situations both compatible with the same state of affairs. In a multi-valued setting, an argument always holds partially, *always* because there is no probabilistic uncertainty involved and *partially* because it can be experienced to different degrees. However, at the same time this is also true for the negation of the claim. Going back to the tomato, the tomato is rotten, but maybe *not so rotten* that parts of it cannot be eaten. Given n arguments with vague claims, there are again 2^n ways in which the set of arguments can partially satisfy the same state of affairs, each situation with a degree of truth associated with it. In each situation we consider the degree to which some arguments satisfy the state of affairs and the others do not satisfy it.

2.1 Computing A_{IN}, $\mu_{A_{IN}}$, $P_{A_{IN}}$

Both $PAFs$ and $MVAFs$ need to compute A_{IN}, the set of subgraphs where a is labelled *in*. The original way of computing $P_{A_{IN}}$ in a PAF is the *constellation approach* proposed in [3], that collects all the subgraphs where a is label in and it computes $P_{A_{IN}}$ by adding up the probability of each subgraph. An equivalent but more efficient way of finding A_{IN} is to apply the recursive algorithm 1 (proposed in [11]), that directly maps the conditions of theorem 1 (see [11] for a comprehensive explanation).

Algorithm 1. A is a node, L a label, P is the list of parent nodes of A.

```
FindSet(A,L,P):
if A in P:
   return empty_set //cycle found
if L = IN:           // Label in
   if A terminal:
      return a //terminal condition for in label
   else:
      add A to P
      for each child C of A
         Cset = Cset AND FindSet(C,OUT,P) //all attackers must be out
return (a AND Cset)
if L = OUT:        // Label out
   if A terminal:
      return NOT(a) //terminal condition for out label
   else
      add A to P
      for each child C of A
   Cset = Cset OR FindSet(C,IN,P) //at least one attacker in
      return (NOT(a) OR Cset)
```

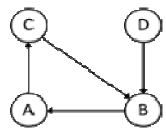


Fig. 2. An argumentation graph.

PAFs and MVAFs differ in the way $P_{A_{IN}}$ and $\mu_{A_{IN}}$ are computed, a direct consequence of the different properties of probability versus multi-valued logics operators. In a MVAF the $x \otimes \ominus x = 0$ (excluded middle), $x \oplus \ominus x = 1$ and the distributive property of $\otimes$ w.r.t. $\oplus$ are not always verified. This implies that the expressions generated by algorithm 1 and by the constellation approach might generate different values. We preferred to use algorithm 1 for computing an expression of A_{IN} suitable for $MVAFs$. As explained in [11], the choice is justified by the fact that algorithm 1 is a recursive algorithm that directly maps the complete labelling conditions found in theorem 1 and, since it performs a tree visit, it does not lose information about the topology of the graph (unlike the constellation approach).

Unlike algorithm 1, the *constellation approach* generates expressions of A_{IN} containing redundant arguments that have no effect on the label of argument a, generating redundant longer conjunctive expressions, resulting in degrees of truth decreasing rapidly and leading the reinstatement property occur only rarely. However, algorithm 1 should be applied respecting the properties of $MVAFs$ when we simplified its output.

Another advantage of algorithm 1 is that, when applied to a $MVAF$, the computation of degrees of truth requires a polynomial time. In fact, unlike probability or possibility calculus the three multi-valued logics proposed have truth-functional operators, i.e. the degree of truth of an expression is fully determined by the degree of truth of its components (as long as no uncertainty is involved in the gradual properties). Therefore degrees of truth can be computed during the recursive visit of algorithm 1, while in the probabilistic case $P_{A_{IN}}$ is obtained only when an expression for A_{IN} has been obtained.

Example 2. Referring to figure 2, a is labelled *in* when:

$$A_{IN} = AB_{OUT} = A(\bar{B} + D_{IN} + C_{IN}) = A(\bar{B} + D + CA_{OUT}) = A(\bar{B} + D).$$

Note how CA_{OUT} identifies a cycle and returns the empty set. In a PAF with $P_A = 0.9, P_B = 0.8, P_D = P_C = 0.7$, we rewrite the expression of A_{IN} using the two disjoint sets $A\bar{B} + ABD$, it is $P(A_{IN}) = P_A(1 - P_B) + P_A P_B P_C = 0.553$. In a $MVAF$ with $\mu_A = 0.9$, $\mu_B = 0.8, \mu_C = \mu_D = 0.7$it is $\mu_{A_{IN}} = \mu(A \otimes (\bar{B} \otimes D)) = \min(\mu_A, \max(1 - \mu_B, \mu_D)) = 0.7$

Table 2. Some properties of MVAF ($0 < \mu < 1$)

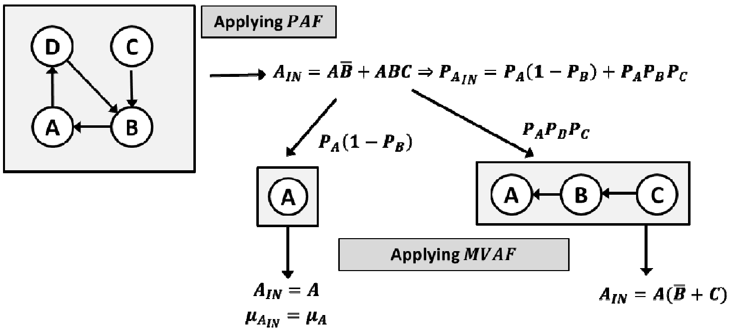
	Property	P	Z	G	L		Property	P	Z	G	L
1	$\mu_{A_{IN}} \le \mu_A$	-	+	+	+	**6**	$\mu_{A_{IN}} + \mu_{A_{OUT}} = 1$ *when* $\mu_{A_U} = 0$	+	+	-	-
2	$\mu_{A_{IN}} < \mu_A$	+	-	-	-	**7**	If one attacker b^i of a has $\mu_{B^i{}_{IN}} >$ 0.5 then $\mu_{A_{IN}} < 0.5$	+	+	+	+
3	$\mu_{A_U} + \mu_{A_{IN}} + \mu_{A_{OUT}} = 1$	+	-	-	-	**8**	Can a be totally defeated?	-	-	+	+
4	$\mu_{A_U} + \mu_{A_{IN}} + \mu_{A_{OUT}} \le 1$	+	-	+	-	**9**	If $\mu_{A_{IN}} < \mu_A$, can a be always fully reinstated by adding new arguments?	-	+	+	+
5	$\mu_{A_{IN}} + \mu_{A_{OUT}} \le 1$	+	+	+	-	**10**	Accrual of attacks?	+	-	-	+

2.2 Properties

We report some of the properties of MVAF, and we check if the properties identified for the probabilistic case hold in this setting. A property similar to the rationality property introduced by Hunter in [4] for the probabilistic case (property 7) holds, while the fundamental relation (property 3) does not hold, even if weaker versions (properties 4,5 and 6) still hold in the multi-valued case. In table 2, we assume $0 < \mu < 1$. For the probabilistic case, it is obviously $P_{A_{IN}}, P_A$ rather than $\mu_{A_{IN}}, \mu_A$. P refers to the probabilistic case, while Z, G, L refer to the three multi-valued systems considered.

3 PROBABILISTIC AND GRADUAL ARGUMENTATION

In a dialectical process, probabilistic and/or vague arguments co-exist. In order to handle this situation, we propose a layered approach where we first perform a probabilistic analysis and then we deal with vagueness, treating each system with its own properties. In the probabilistic case, a subgraph represents a situation where some arguments hold in the dialectical process and others do not hold, while in the multi-valued case all the arguments "hold" but only partially. Let us consider the graph of example 2 with $P(A) = 0.9, P(B) = 0.8, P(C) = P(D) = 0.7$ and $\mu_A = 0.9, \mu_B = 0.8, \mu_C = \mu_D = 0.7$. We wonder to which probability and degree of truth we can accept a. We first deal with probability, obtaining a list of set of subgraphs where the label *in* holds, each set of subgraphs with a probability associated. Then we apply the multi-valued analysis to each set of subgraphs (found in the probabilistic case) separately to obtain a degree of truth for a in each of them.



	$P_{A_{IN}}$	$\mu_{A_{IN}}$
	$P_A(1 - P_B) = 0.18$	$\mu\_A = 0.8$
	$P_A P_B P_C = 0.504$	$\mu_{A_{IN}} = \min(\mu_A, \max(1 - \mu_B, \mu_C)) = 0.7$

Figure 3. Mixing Probabilistic and Vague Arguments

As shown in figure 3, we first apply algorithm 1 to get $A_{IN} = A\bar{B} + ABC$ (we are allowed to simplify since we are dealing with the probabilistic case). The two clauses are two <u>distinct</u> situations to be further studied, and they are the only two situations, since all the other subgraphs have been discarded by the probabilistic layer and therefore it makes no sense to consider their degree of truth. The first clause $A\bar{B}$ is a set of subgraphs where a is present, b is not present and c is disconnected, i.e. it contains argument a only. The probability of $A\bar{B}$ is $P(A\bar{B}) = 0.8 * (1 - 0.7) = 0.24$. The degree of truth of this situation is the degree of truth of argument a on its own, $\mu_A = 0.8$. The second term ABC identifies a single subgraph with all the arguments present. Its probability is $0.9 * 0.8 * 0.7 = 0.504$. What is the value of μ_A in this situation? We have all the arguments present, so we apply algorithm 1 to obtain $A_{IN} = A(\bar{B} + C)$ that with Zadeh's logic is equal to 0.6. Therefore argument a is labelled *in* with probability 0.18 and degree of truth 0.9, or with probability 0.504 with a degree of truth of 0.7.

This example shows how algorithm 1 can be used as a generic computational framework for dealing with gradualism and uncertainty in abstract argumentation.

4 RELATED WORKS

Conceptually, our framework is closer to the work done in the context of probabilistic argumentation frameworks. The idea of merging probabilities and abstract argumentation was first presented by Dung [2], and a more detailed formalization was provided by Li [3], along with works by Hunter [4] and Thimm [7]. [3] introduces the notion of a constellation approach. [7] and [4]'s epistemic approach start from a complementary angle.

Both authors assume that there is already an uncertainty measure – potentially not probabilistic – defined on the admissibility set of each argument, and they study which properties this uncertainty measure should satisfy in order to be rational. Regarding works that explicitly define fuzzy argumentation systems, we should mention the framework by Janssen [8], where fuzzy labels may be interpreted as fuzzy membership to an extension. However, [8]'s approach differs significantly from ours in that the attack relation that defines the framework is taken to be fuzzy and the conflict-free and admissibility definitions are changed accordingly. In [9] a certitude factor is added to the labels *in*, *out* and *undec*, as it is in our work as well. The work proposes an *equational* approach to abstract argumentation, where the degrees of arguments have to satisfy a set of properties modelled as equations, properties that might not have any link to a fuzzy logic system. On the contrary, our computation of degrees of truth is a more consistent approach, exploiting both argumentation semantics and multi-valued logics.

5 CONCLUSIONS

In this paper we progressed further our analysis of multi-valued argumentation frameworks ($MVAFs$). $MVAFs$ are an extension of Dung's abstract argumentation, where arguments have a degree of truth associated with them. We proposed a list of properties of $MVAFs$ considering the major multi-valued logics. We also proposed a computational framework that joins multi-valued and probabilistic argumentation frameworks to handle situations affected by gradualism and probabilistic uncertainty. Our work is an initial proposal in the direction of an argumentation framework able to handle multiple sources of uncertainty. Future works will consider the properties of the proposed framework, the meaning of its output and its extension to other complete semantics and sources of uncertainty such as possibility theory.

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