Resolution-Based Grounded Semantics Revisited

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Abstract. The resolution-based grounded semantics constitutes one of the most interesting approaches for the evaluation of abstract argumentation frameworks. This particular semantics satisfies a large number of desired properties, among them all properties proposed by Baroni and Giacomin. In recent years, the analysis of argumentation semantics has been extended by further topics, among them characterizations for equivalence notions, intertranslatability issues, and expressibility in terms of signatures (all possible sets of extensions a semantics is capable to express). In this line of research, resolution-based grounded semantics has been neglected so far. We close this gap here, compare the expressibility of resolution-based grounded semantics with other prominent semantics, provide a characterization for strong equivalence and complement existing complexity results.

Keywords. abstract argumentation, resolution-based grounded semantics, complexity, intertranslatability, signatures, strong equivalence

1. Introduction

Since the proposal of *abstract argumentation frameworks* (AFs) as a formalism to model and solve argumentation problems [1], there has been vital research on various *semantics* thereof. Besides the introduction of further semantics (e.g. [2,3]) there have been some efforts to investigate their "adequacy". In particular, Baroni and Giacomin [4] provided a comprehensive collection of desirable properties (so-called *principles*) for argumentation semantics in order to rely on formal criteria rather than basic intuitions for the definition of semantics. The first, and to the best of our knowledge only, published semantics to satisfy all properties from [4] is the *resolution-based grounded* semantics [5].

More recently, further aspects for a systematic investigation of argumentation semantics have been proposed. First, a comparison of their expressiveness has been carried out by studying *intertranslatability* issues [6,7]. That is, assuming arbitrary semantics σ and τ, if there is an exact translation of any AF such that τ-extensions of the transformed AF coincide with the σ-extensions of the original AF, then τ can be rated as at least as expressive as σ. Moreover, the modelling capabilities of argumentation semantics have been studied in terms of *signatures* [8]. For a semantics σ, its signature is simply the collection of all possible sets of extensions that can be obtained by AFs with semantics σ. Finally, the notion of *strong equivalence* [9,10] has lead to a further categorization of different semantics in terms of kernels. Roughly speaking, the kernel of an AF is obtained by removing all redundant attacks (where redundancy means that the attack does not play a role for the computation of the extensions, no matter how the entire AF looks like). In all these works, resolution-based grounded semantics has been neglected so far. We close this gap here with the following contributions:

• First, we add to the *complexity* landscape of resolution-based grounded semantics complementing results from [5] by showing P-hardness for the verification problem as well as NP-hardness of credulous acceptance for the special case of bipartite AFs.
• Then we give results approximating the signature of the resolution-based grounded semantics by relating it to the signatures of stable and preferred semantics, respectively. In particular, we provide some necessary conditions which are rather different to other semantics. Basically, the conditions rely on a generalisation of the observation that there is no AF which exactly has $\{a\}$, $\{b\}$ and $\{c\}$ as its resolution-based grounded extensions.
• Concerning intertranslatability, we provide an efficient translation from the grounded to the resolution-based grounded semantics and we show that no (efficient) translations between resolution-based grounded semantics on the one side and stable, admissible and complete semantics on the other side exist.
• Our final result states that resolution-based grounded semantics and grounded semantics possess the same kernels. This implies that two AFs are strongly equivalent under resolution-based grounded semantics iff they are strongly equivalent under the grounded semantics. This is also interesting from the point of view that all other semantics require other forms of kernels.

2. Preliminaries

We consider a fixed countable set $\mathfrak{A}$ of arguments. An argumentation framework (AF) is a pair (A,R) where $A \subseteq \mathfrak{A}$ is a finite set of arguments and $R \subseteq A \times A$ represents the attack-relation. For an AF $F = (B,S)$ we use A_F to refer to B and R_F to refer to S. For an AF F, $a,b \in A_F$ and $S,T \subseteq A_F$ we further write $a \mapsto_F b$ for $(a,b) \in R_F$, $S \mapsto_F b$ for $\exists s \in S : (s,b) \in R_F$, $a \mapsto_F T$ for $\exists t \in T : (a,t) \in R_F$, and $S \mapsto_F T$ for $\exists s \in S \exists t \in T : (s,t) \in R_F$. An AF F is *symmetric* if $\forall (a,b) \in R_F : (b,a) \in R_F$ and *loop-free* if $\nexists a \in A_F : (a,a) \in R_F$. The *union* of AFs F_1 and F_2 is defined as $F_1 \cup F_2 = (A_{F_1} \cup A_{F_2}, R_{F_1} \cup R_{F_2})$.

For an AF $F = (A,R)$ and $S \subseteq A$, we say that S is *conflict-free* in F if there are no $a,b \in S$ such that $(a,b) \in R$; $a \in A$ is *defended* by S in F if for each $b \in A$ with $(b,a) \in R$, $S \mapsto_F b$. We further define $S^{\oplus}_F = \{s \in A \mid S \mapsto_F s\}$ and $S^{+}_F = S \cup S^{\oplus}_F$. The part of F not "covered" by S is denoted as $cut_S(F) = (B, R_F \cap (B \times B))$ where $B = (A_F \setminus S^{+}_F)$. Semantics map AFs (A,R) to collections $\mathcal{S} \subseteq 2^A$ of sets of arguments, the so-called *extensions*.

Definition 1. Let $F = (A,R)$ be an AF and $S \subseteq A$ such that S is conflict-free in F.

- S is a *stable* extension of F, i.e., $S \in stb(F)$, if for each $a \in A \setminus S$, $S \mapsto_F a$.
- S is an *admissible* extension of F, i.e., $S \in adm(F)$, if each $a \in S$ is defended by S.
- S is a *preferred* extension of F, i.e., $S \in pref(F)$, if $S \in adm(F)$ and for each $T \in adm(F)$, $S \not\subset T$.
- S is a *complete* extension of F, i.e., $S \in com(F)$, if $S \in adm(F)$ and for each $a \in A$ defended by S in F, $a \in S$ holds.

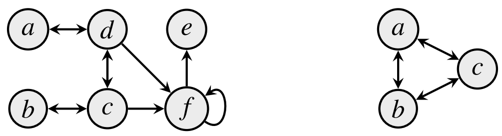

Figure 1. Example argumentation frameworks F (left) and F' (right).

- S is a *grounded* extension of F, i.e., $S \in grd(F)$, if $S \in com(F)$, and for each $T \in com(F)$, $T \not\subset S$.

It is well known that for $\sigma \in \{adm, pref, com, grd\}$, and any AF F, $\sigma(F) \neq \emptyset$; only stable semantics may yield an empty set of extensions. Moreover, $grd(F)$ contains exactly one extension, and we subsequently identify – with some abuse of notation – this single extension via $grd(F)$. The grounded extension of an AF F can also be given by the least fixed point of the *characteristic function* $\mathscr{F}_F : 2^{A_F} \to 2^{A_F}$, where $\mathscr{F}_F(S) = \{a \in A_F \mid a \text{ is defended by } S \text{ in } F\}$.

The family of *resolution-based* semantics [5] is a parametric approach defined in the following way: given an AF $F = (A,R)$, a resolution[1] of F is any AF (A,R') with $R' \subseteq R$, such that each $(a,a) \in R$ is also contained in R' and for each $(a,b) \in R$ with $a \neq b$ either $(a,b) \in R'$ or $(b,a) \in R'$, but not both. We denote the set of all resolutions of F as $\gamma(F)$. If σ is an extension-based semantics, the *resolution-based* σ semantics, σ^*, is given through $\sigma^*(F) = \min\left(\bigcup_{F' \in \gamma(F)} \sigma(F')\right)$ where min is with respect to $\subseteq$.

Example 1. Consider the AF F from Figure 1. We have $pref(F) = \{\{a,b\}, \{a,c,e\}, \{b,d,e\}\}$, $stb(F) = \{\{a,c,e\}, \{b,d,e\}\}$ and $grd(F) = \emptyset$. For computing $grd^*(F)$, we need to check eight resolutions of F and finally get $grd^*(F) = \{\{a,b\}, \{a,c,e\}, \{b,d,e\}\}$. We recall at this place that the resolution-based grounded semantics obviously is *multiple-status*. Here grd^* and $pref$ coincide, but this is not always true. For instance, for the AF F' from Figure 1, $pref(F') = \{\{a\}, \{b\}, \{c\}\}$ while $grd^*(F') = \{\emptyset\}$.

3. Complexity Analysis

We start with a few new complexity results.[2] Baroni et al. [5] already have given a thorough complexity analysis in terms of resolution-based grounded semantics. In particular, for the prominent decisions problems

- $Cred_\sigma$: given AF $F = (A,R)$, $a \in A$, does there exist $E \in \sigma(F)$ s.t. $a \in E$;
- $Skept_\sigma$: given AF $F = (A,R)$, $a \in A$, does $a \in E$ hold for all $E \in \sigma(F)$;
- Ver_σ: given AF $F = (A,R)$, $S \subseteq A$, does $S \in \sigma(F)$ hold;

(σ denotes a semantics), they have shown NP-completeness for $Cred_{grd^*}$, coNP-completeness for $Skept_{grd^*}$, and membership in P for Ver_{grd^*}. We first strengthen the result for Ver_{grd^*} by giving a matching hardness proof. This result will be useful later to show that certain translations between semantics are impossible under some standard complexity-theoretic assumptions.

[1] We use a slightly different presentation compared to [5].

[2] These results are from the PhD Thesis [11] of the first named author but are not published elsewhere.

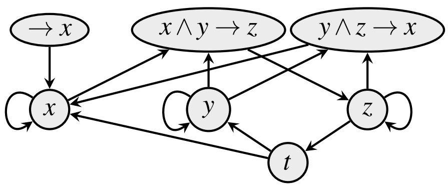

Figure 2. Argumentation framework $F_{\varphi,z}$ for $\varphi = \{\rightarrow x, x \wedge y \rightarrow z, y \wedge z \rightarrow x\}$.

Proposition 1. *The problem Ver_{grd^*} is P-complete.*

Proof. For the missing hardness we give a reduction from the well-known P-hard problem HORNSAT to Ver_{grd^*}. To this end, we define for any pair (φ, z) where $\varphi = \{r_l : b_{l,1} \wedge \cdots \wedge b_{l,i_l} \rightarrow h_l \mid 1 \leq l \leq n\}$ is a definite Horn formula over atoms X and $z \in X$, an AF $F_{\varphi,z} = (A, R)$ with $A = \varphi \cup X \cup \{t\}$ and R as follows:

$$R = \{(x,x),(t,x) \mid x \in X \setminus \{z\}\} \cup \{(z,t)\} \cup \{(r_l,h_l),(b_{l,j},r_l) \mid r_l \in \varphi, 1 \leq j \leq i_l)\}$$

where $t \notin X$ is a fresh argument. Figure 2 shows an example. AF $F_{\varphi,z}$ can be constructed using only logarithmic space in the size of φ. Moreover, we can assume that for all rules $r_l \in \varphi$ the head $\{h_l\}$ and body $\{b_{l,i}\}_{1 \leq i \leq i_l}$ are disjoint without affecting the P-hardness of HORNSAT. Then, there are no symmetric attacks in $F_{\varphi,z}$. Hence $grd(F_{\varphi,z}) = grd^*(F_{\varphi,z})$ and thus Ver_{grd^*} for such AFs boils down to the problem of Ver_{grd}. It only remains to show that z is in the minimal model of φ iff $grd(F_{\varphi,z}) = \{\varphi \cup \{t\}\}$. This follows immediately by the hardness result for *grd* in [6]. □

Our second result concerns bipartite AFs. This class was first discovered by Dunne [12] who showed that problems $Cred_\sigma$ and $Skept_\sigma$ sometimes become easier for bipartite AFs. In fact, Baroni et al. [5] showed tractability for $Skept_{grd^*}$ on bipartite AFs, while the complexity for $Cred_{grd^*}$ remained open for that class of AFs. However, for a more general version of this problem (given AF $F = (A,R)$, $S \subseteq A$, does there exist $E \in \sigma(F)$ s.t. $S \subseteq E$) they proved NP-completeness. In what follows, we show that already the standard $Cred_{grd^*}$ problem remains NP-complete for bipartite AFs.

Proposition 2. *The problem $Cred_{grd^*}$ is NP-complete even for bipartite* AF*s.*

Proof. The membership is immediate via the results for the more general case in [5]. We prove hardness by a reduction from the Monotone SAT problem. Thus let $\varphi = \bigwedge_{c \in C} c$ be a monotone CNF over atoms X and (C_p, C_n) a partition of C in positive clauses C_p and negative clauses C_n. We define the following AF $F_\varphi = (A,R)$, where $A = \{t\} \cup C \cup X \cup \bar{X} \cup \{a_c, b_c, d_c, e_c \mid c \in C_n\}$ and $R = \{(c,t) \mid c \in C\} \cup \{(x,\bar{x}),(\bar{x},x) \mid x \in X\} \cup \{(l,c) \mid \text{literal } l \text{ occurs in } c \in C_p\} \cup \{(l,d_c) \mid \text{literal } l \text{ occurs in } c \in C_n\} \cup \{(a_c,c),(a_c,b_c),(b_c,a_c),(d_c,b_c),(a_c,e_c),(e_c,d_c) \mid c \in C_n\}$. The reduction is illustrated in Figure 3. We can partition the arguments A in two independent sets, i.e. in the sets $X \cup \{a_c, d_c, t \mid c \in C_n\}$ and $\bar{X} \cup \{b_c, e_c, \mid c \in C_n\} \cup C$. Hence F_φ is a bipartite AF. We show that φ is satisfiable iff t is credulously accepted in F_φ.

We start with some observations on F_φ. When resolving a symmetric attack between an $x \in X$ and $\bar{x} \in \bar{X}$ we choose either x or $\bar{x}$ for being in the grounded extension of the resolved AF. Thus for two resolutions G, G' that such that $(x,\bar{x})$ is an attack in G and $(\bar{x},x)$ in G' we have that the corresponding grounded extensions of the resolved AFs are

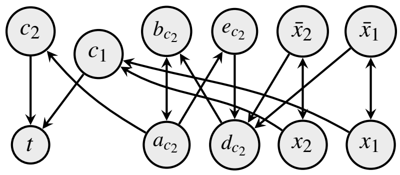

Figure 3. F_φ from the proof of Proposition 2, for formula $\varphi = c_1 \wedge c_2$ with $c_1 = x_1 \vee x_2$ and $c_2 = \neg x_1 \vee \neg x_2$.

clearly not in a $\subseteq$-relation. So to prove that the grounded extension of a resolved AF is also a resolution-based grounded extension of F_φ, we only have to consider resolutions which make the same choice on the arguments $X \cup \bar{X}$.

$\Rightarrow$: Given model $M \subseteq X$ satisfying φ. Let us consider the resolutions G such that $M \cup (\overline{A \setminus M}) \subseteq grd(G)$. As M is a model we have that each argument $c \in C_p$ is attacked by M in G and for each $c \in C_n$ the argument d_c is attacked by $\overline{A \setminus M}$. Now for each $c \in C_n$ we have to resolve the attacks $(a_c, b_c), (b_c, a_c)$, and dependent on the choice we either get $a_c \in grd(G)$ or $\{b_c, e_c, c\} \subset grd(G)$. As these sets are not in $\subseteq$-relation, both give rise to different resolution-based grounded extensions of F_φ. Now let us consider the resolution G such that for each $c \in C_n$, $a_c \in grd(G)$. Then, the argument t is defended in G and hence $t \in grd(G)$. Now as $grd(G)$ is $\subseteq$-minimal and $t \in grd(G)$ we have that t is credulously accepted.

$\Leftarrow$: Let φ be unsatisfiable and towards a contradiction assume there is an $E \in grd^*(F_\varphi)$ with $t \in E$. Thus there exists a resolution G of F_φ such that $E = grd(G)$. By similar arguments as above, it is obvious that G has to contain attacks $(x, \bar{x})$ for each $x \in E$ and attacks $(\bar{x}, x)$ for each $\bar{x} \in E$. Thus only two choices for G remain, namely G_1 with attack (a,b) and G_2 with attack (b,a). As $E \cap X$ is not a model of φ, there exists a $c \in C$ such that M does not satisfy c. If $c \in C_p$ then by construction E does not attack c and hence t is not defended, a contradiction for E being grounded extension of G_1 or G_2. Now let us consider the case where $c \in C_n$. Then d_c is not attacked by E and thus $grd(G_2) \cap \{a_c, b_c, d_c, e_c\} = \emptyset$. It follows that $t \notin grd(G_2)$. Finally, it can be checked that in the case that d_c is not attacked by E, $grd(G_1) \supset grd(G_2)$. But then, $E \notin grd^*(F_\varphi)$. □

4. Expressibility

In order to study the expressibility of different semantics, two concepts have been introduced in the literature. First, so-called signatures characterize the sets of extensions that can be realized by a specific semantics. Second, translations study how the concept of one semantics can be translated, by modifying the AF, into another semantics.

4.1. Signatures

In order to compare the expressibility of different argumentation semantics [8] has introduced signatures, $\Sigma_\sigma = \{\sigma(F) \mid F \text{ is an AF}\}$, where σ is a semantics. In what follows we want to compare Σ_{grd^*} with Σ_{pref} and Σ_{stb}. The latter two have been exactly characterized in [8]. In order to review these results we need some formal concepts.

Given $\mathbb{S} \subseteq 2^{\mathfrak{A}}$, $Args_\mathbb{S}$ denotes $\bigcup_{S \in \mathbb{S}} S$ and $Pairs_\mathbb{S}$ denotes $\{(a,b) \mid \exists S \in \mathbb{S} : \{a,b\} \subseteq S\}$. $\mathbb{S}$ is called an *extension-set* (over $\mathfrak{A}$) if $Args_\mathbb{S}$ is finite. $\mathbb{S}$ is called (i) incomparable if all elements $S \in \mathbb{S}$ are pairwise incomparable, i.e. for each $S, S' \in \mathbb{S}$, $S \subseteq S'$ implies $S = S'$; (ii) tight[3] if for all $S \in \mathbb{S}$ and $a \in (Args_{\mathbb{S}} \setminus S)$ there exists an $s \in S$ such that $(a,s) \notin Pairs_{\mathbb{S}}$; (iii) adm-closed if for each $A, B \in \mathbb{S}$ the following holds: if $(a,b) \in Pairs_{\mathbb{S}}$ for each $a, b \in A \cup B$, then also $A \cup B \in \mathbb{S}$. In [8], the following characterizations have been shown:

$$\Sigma_{stb} = \{\mathbb{S} \mid \mathbb{S} \text{ is an incomparable tight extension-set}\};$$

$$\Sigma_{pref} = \{\mathbb{S} \neq \emptyset \mid \mathbb{S} \text{ is an incomparable adm-closed extension-set}\}.$$

Note that $(\Sigma_{stb} \setminus \{\emptyset\}) \subset \Sigma_{pref}$. Example 1 already described an AF whose resolution-based grounded extensions cannot be expressed by stable semantics, implying the following:

Proposition 3. $\Sigma_{grd^*} \not\subseteq \Sigma_{stb}$.

Proof. For the AF F from Example 1, $grd^*(F) = \{\{a,b\},\{a,c,e\},\{a,d,e\}\}$, thus $grd^*(F) \in \Sigma_{grd^*}$. However, $grd^*(F)$ is not tight, since for $\{a,b\}$ and e it holds that (a,e) and (b,e) is contained in $Pairs_{grd^*(F)}$. Thus, $grd^*(F) \notin \Sigma_{stb}$. □

Hence, there exists an AF F such that $grd^*(F) \neq stb(F')$ holds for all possible AFs F'. In other words, the grd^* semantics is capable of realizing extension-sets which are not realizable by stable semantics (or weaker semantics as stage or naive). Next, we will show that in contrast, $\Sigma_{grd^*} \subseteq \Sigma_{pref}$ holds.

To this end we recall some definitions and results from [5], using slightly different notation though. Given an AF F, the set of *initial SCCs* $\mathfrak{I}(F)$ is the set of AFs associated to the strongly connected components of the graph underlying F, which are not attacked by an argument of any other SCC. It holds that $\forall I, J \in \mathfrak{I}(F), I \neq J : A_I \not\mapsto_F A_J$. Further for $\mathfrak{I} \subseteq \mathfrak{I}(F)$ we define the AF $F^{\mathfrak{I}} = \bigcup_{I \in \mathfrak{I}} I$. The following lemma containing an alternative, recursive definition of grd^* is immediate by Theorem 2 and Lemma 9 of [5]:

Lemma 1. *Given an* AF F, *if* $grd^*(F) \neq grd(F)$ *then there is a non-empty set* $\mathfrak{I} \subseteq \mathfrak{I}(cut_{grd(F)}(F))$ *of initial SCCs of* $cut_{grd(F)}(F)$ *such that (i) each* $I \in \mathfrak{I}$ *is loop-free, symmetric, and the underlying undirected graph is acyclic; (ii)* $S \in grd^*(F)$ *iff* $S = (T \cup U \cup V)$ *where* $T = grd(F)$, $U \in stb(cut_T(F)^{\mathfrak{I}})$, *and* $V \in grd^*(cut_{(T \cup U)}(F))$.

The following observation is important.

Lemma 2. *Given* $F = (A,R)$ *and sets* $S_1 \neq S_2$. *Then,* $S_1, S_2 \in grd^*(F)$ *implies* $S_1 \mapsto_F S_2$.

Proof. Consider an AF F with $S_1, S_2 \in grd^*(F)$ such that $S_1 \neq S_2$. Let $T = grd(F)$, $\mathfrak{I}$ as in Lemma 1 and $F' = cut_T(F)^{\mathfrak{I}}$. Since obviously $grd(F) \neq grd^*(F)$ we follow by Lemma 1 that, for $i \in \{1,2\}$, $T \subset S_i$ and $\exists U_i \in stb(F') : U_i \subseteq S_i$. If $U_1 \neq U_2$ we are done because U_1 must attack all arguments in $U_2 \setminus U_1$. On the other hand if $U_1 = U_2$, let $F'' = cut_{(T \cup U_1)}(F) = cut_{(T \cup U_2)}(F)$. We have $(S_i \setminus (T \cup U_i)) \in grd^*(F'')$ and we can reason as above. Since F is finite, the result follows by induction. □

Proposition 4. $\Sigma_{grd^*} \subseteq \Sigma_{pref}$.

Proof. For any AF F, $grd^*(F)$ is by definition an incomparable and non-empty extension-set. It remains to show that $grd^*(F)$ is adm-closed. By Lemma 2, for any distinct $S_1, S_2 \in grd^*(F)$, $S_1 \mapsto_F S_2$ holds. Hence $\exists s_1, s_2 \in (S_1 \cup S_2) : (s_1, s_2) \notin Pairs_{grd^*(F)}$. □

[3] We give here a slightly simpler definition for tight, which does not affect the forthcoming result for Σ_{stb}.

The following results concerning realizability show certain and severe limits of the expressibility the resolution-based grounded semantics suffers from.

Proposition 5. *Let F be an* AF *and $S \subset A_F$. There are no pairwise disjoint sets S_1, S_2, S_3 such that $\{(S \cup S_1), (S \cup S_2), (S \cup S_3)\} \subseteq grd^*(F)$.*

Proof. Consider pairwise disjoint sets S_1, S_2, S_3 such that $\{(S \cup S_1), (S \cup S_2), (S \cup S_3)\} \subseteq grd^*(F)$. Let $T = grd(F)$. By Lemma 1, there exists $\mathfrak{I} \subseteq \mathfrak{I}(cut_T(F))$ such that for $F' = cut_T(F)^{\mathfrak{I}}$ and $i \in \{1,2,3\}$ it holds that $T \subset S$ and $\exists U_i \in stb(F') : U_i \subseteq (S \cup S_i)$. Note that each U_i has full range in F' (i.e. $(U_i)^+_{F'} = A_{F'}$), hence $(A_{F'} \setminus U_i) \cap S_i = \emptyset$. Therefore and since the elements of $stb(F')$ are incomparable, either $U_1 = U_2 = U_3 \subseteq S$ or all U_i are pairwise different. In the latter case each $u \in (U_i \setminus S)$ must be attacked by each $U_j \setminus S$ $(i \neq j)$, a contradiction to the undirected variant of F' being acyclic (cf. Lemma 1). In the former case $(U_1 = U_2 = U_3 \subseteq S)$ let $F'' = cut_{(T \cup U_i)}(F)$ for any $i \in \{1,2,3\}$, and $S' = S \setminus (T \cup U_i)$, then $\{(S' \cup S_1), (S' \cup S_2), (S' \cup S_3)\} \subseteq grd^*(F'')$. Hence we can follow the same reasoning as above and, since F is finite, the result follows by induction. □

This already suggests quite strong limitations concerning the structural diversity of extension-sets under the resolution-based grounded semantics.

Corollary 1. *Let $\mathbb{S}$ be an extension-set containing three pairwise disjoint sets S_1, S_2, and S_3. There is no* AF *F such that $grd^*(F) \supseteq \{S_1, S_2, S_3\}$.*

Corollary 2. *Given an* AF *$F = (A,R)$ and arguments $a, b \in A$ such that $\{a\}, \{b\} \in grd^*(F)$. Then, $grd^*(F) = \{\{a\}, \{b\}\}$.*

Proof. If $\{a\}, \{b\} \in grd^*(F)$ then any further extension would have to be either disjoint or not incomparable to $\{a\}$ and $\{b\}$, both contradictions to previous observations. □

As a simple consequence, our results show that there is no AF F, such that $grd^*(F) = \{\{a\}, \{b\}, \{c\}\}$. Note that many semantics are indeed able to express this extension-set, in particular, stable and preferred semantics when applied to a clique $\{a, b, c\}$.

Proposition 6. $\Sigma_{grd^*} \subset \Sigma_{pref}$ *and* $\Sigma_{stb} \not\subseteq \Sigma_{grd^*}$.

We leave an exact characterization of Σ_{grd^*} for future work. Finally, given the relations between signatures provided in [8] the results of this section also extend to semantics beyond the scope of the paper, e.g. given that $\Sigma_{pref} = \Sigma_{sem}$ we have $\Sigma_{grd^*} \subset \Sigma_{sem}$[4].

4.2. Intertranslatability

The concept of translations between AFs was studied in [6,7]. The underlying idea is to modify an AF F to an AF F' such that the extensions of F wrt. a semantics σ correspond to the extensions of F' wrt. a different semantics τ.

Definition 2. A function *Tr* mapping AFs to AFs is a *translation*[5] from semantics σ to semantics τ if there exists a finite set $\mathscr{R}$ such that $\sigma(F) = \tau(Tr(F)) \setminus \mathscr{R}$ holds for each AF F. If $Tr(F)$ can be computed using only logarithmic space we call *Tr efficient*. Furthermore if $F \subseteq Tr(F)$, i.e. $A_F \subseteq A_{Tr(F)}$ and $R_F \subseteq R_{Tr(F)}$ for each F, *Tr* is *covering*.

[4] Semi-stable semantics: $S \in sem(F)$ iff $S \in adm(F)$ and $\forall T \in adm(F) : S^+_F \not\subset T^+_F$.

[5] This corresponds to what is called a "weakly exact translation" in [6].

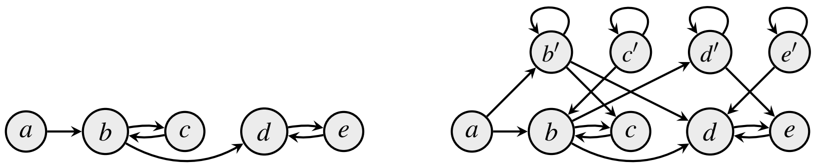

Figure 4. Illustration of Tr_α, with the original AF on the left side and the transformed AF on the right side.

There is a close connection between signatures and translations. If for two semantics σ, τ it holds that $\Sigma_\sigma \subseteq \Sigma_\tau$ then there exists a translation from σ to τ with $\mathscr{R} = \emptyset$. However, it does not tell us anything about the properties of this translation. In particular, in the case that $\Sigma_\sigma \subseteq \Sigma_\tau$ one interesting questions is whether there is an *efficient* translation from σ to τ. On the other hand, the set $\mathscr{R}$ gives a tool to translate semantics σ, τ also in the case where $\Sigma_\sigma \not\subseteq \Sigma_\tau$. However the results in this section show that this is not significant for resolution-based grounded semantics. That is, whenever $\Sigma_\sigma \not\subseteq \Sigma_\tau$ we show that there is no translation from σ to τ.

We first give an efficient translation from grounded to resolution-based grounded semantics, which also covers the original framework (see also Figure 4).

Translation 1. The translation $Tr_\alpha(F)$ is defined as $Tr_\alpha(F) = (A^*, R^*)$ where

$$A^* = A_F \cup A' \quad \text{where } A' = \{a', b' \mid (a,b),(b,a) \in R_F, a \neq b\}$$
$$R^* = R_F \cup \{(a,b') \mid a \in A_F, b' \in A', (a,b) \in R_F, (b,a) \notin R_F\} \cup$$
$$\{(a',b) \mid a,b \in A_F, (a,b) \in R_F\} \cup \{(a',a') \mid a' \in A'\}$$

Proposition 7. *Tr_α is an efficient covering translation from grd to grd*.*

Proof. Clearly Tr_α can be computed in logarithmic space. First we show $grd(Tr_\alpha(F)) = grd(F)$. To this end consider that the grounded extension is the least fixed-point of the characteristic function. We show that for each $a \in A$ and integer k the following holds: (i) $\mathscr{F}_F^k(\emptyset) = \mathscr{F}_{Tr_\alpha(F)}^k(\emptyset)$; (ii) a is attacked by $\mathscr{F}_F^k(\emptyset)$ iff a is attacked by $\mathscr{F}_{Tr_\alpha(F)}^k(\emptyset)$; and (iii) if $a' \in A'$ then a is attacked by $\mathscr{F}_{Tr_\alpha(F)}^k(\emptyset)$ iff a' is attacked by $\mathscr{F}_{Tr_\alpha(F)}^k(\emptyset)$.

As the attacks between arguments in A are the same in F and $Tr_\alpha(F)$ we have that (ii) follows immediately from (i). The proof proceeds by induction on k.

Induction base $k = 1$: By construction $a \in A$ is un-attacked in F iff it is un-attacked in $Tr_\alpha(F)$, hence (i) and also (ii) hold. If an argument a is attacked by $b \in \mathscr{F}_F^1(\emptyset)$ then $(a,b) \notin R$, resp. $(a,b) \notin R^*$. Thus if $a' \in R$ also $(b,a') \in R^*$ and therefore (iii) holds.

For the induction step, assume that (i), (ii) and (iii) hold for an integer k. We show that they also hold for $k+1$. First let us consider (i): For an argument $a \in A_F$ we have that $a \in \mathscr{F}_F^{k+1}(\emptyset)$ iff each $b \in A_F$ such that $(b,a) \in R$ is attacked by $\mathscr{F}_F^k(\emptyset)$. By the induction hypothesis (ii) and (iii) this is equivalent to each $b \in A_{Tr_\alpha(F)}$ such that $(b,a) \in R^*$ is attacked by $\mathscr{F}_{Tr_\alpha(F)}^k(\emptyset)$. The last statement itself holds iff $a \in \mathscr{F}_{Tr_\alpha(F)}^{k+1}(\emptyset)$.

Now let us consider (iii): First let us assume that a is attacked by $\mathscr{F}_F^{k+1}(\emptyset)$. If a is also attacked by $\mathscr{F}_F^k(\emptyset)$ then (iii) follows by the induction hypothesis. Hence let us assume that a is not attacked by $\mathscr{F}_F^k(\emptyset)$. Then we have that a is attacked by a $b \in \mathscr{F}_F^{k+1}(\emptyset)$, such that $(a,b) \notin R$, resp. $(a,b) \notin R^*$ (otherwise b is not defended by $\mathscr{F}_F^k(\emptyset)$). But then we have that $(b,a') \in R^*$ and as (i) holds that a and a' are attacked by $\mathscr{F}_{Tr_\alpha(F)}^{k+1}(\emptyset)$. Now let us assume that a' is attacked by $\mathscr{F}^{k+1}_{Tr_\alpha(F)}(\emptyset)$. By construction we have that also a is attacked by $\mathscr{F}^{k+1}_{Tr_\alpha(F)}(\emptyset)$ and as (i) holds that a is attacked by $\mathscr{F}^{k+1}_{F}(\emptyset)$.

Now we have shown that the characteristic functions coincide in each iteration step and thus the also coincide at the least fixed-point, i.e. the grounded extensions coincide.

Finally we show that each resolution of $Tr_\alpha(F)$ yields $grd(Tr_\alpha(F))$ as grounded extension. Consider a resolution F' of $Tr_\alpha(F)$, we show that the characteristic functions $\mathscr{F}_{Tr_\alpha(F)}$ and $\mathscr{F}_{F'}$ coincide. Let $S \subseteq A_{Tr_\alpha(F)}$ be a set of arguments and $a \in A_{Tr_\alpha(F)}$. We have that $a \in \mathscr{F}_{Tr_\alpha(F)}(S)$ iff for each $b \in A_F$ with $(b,a) \in R_F$, b is attacked by S and if $b' \in A'$ also b' is attacked by S. If $b' \notin A'$ then we have that b is not incident with any symmetric attack and hence S attacks b in $Tr_\alpha(F)$ iff S attacks b in F'. Otherwise if $b' \in A'$ then we have that b' is not incident with any symmetric attack and hence S attacks b' in $Tr_\alpha(F)$ iff S attacks b' in F'. By construction, in both cases, S attacks b' implies that S attacks b'. Hence a is defended by S in $Tr_\alpha(F)$ iff a is defended by S in F'. □

We now turn to the results showing that certain translations are impossible.

Proposition 8. *There is no translation from grd* to σ for $\sigma \in \{grd, stb\}$.*

Proof. The case for *grd* follows directly from the fact that there is always a unique grounded extension. For the case of *stb* consider the AF F from Example 1. We had $grd^*(F) = \{\{a,b\},\{a,c,e\},\{b,d,e\}\}$. As we already observed $grd^*(F)$ is not tight and thus also no superset of it can be tight. Hence we can not translate F to *stb* semantics. □

Proposition 9. *There is no translation from σ to grd* for $\sigma \in \{adm, com, stb\}$.*

Proof. Consider the AF $F = (\{a,b,c\},\{(a,b),(b,a),(b,c),(c,b),(c,a),(a,c)\})$. We have that $\{a\},\{b\},\{c\}$ are admissible, complete and stable extensions and by Proposition 5 cannot be part of any *grd** extension set. □

Proposition 10. *There is no translation from grd* to σ for $\sigma \in \{adm, com\}$.*

Proof. Consider the AF $F = (\{a,b\},\{(a,b),(b,a)\})$ with $grd^*(F) = \{\{a\},\{b\}\}$. Now any AF F' that has $\{a\},\{b\} \in adm(F')$ ($\{a\},\{b\} \in com(F')$) also has $\emptyset \in adm(F')$ ($\emptyset \in com(F')$). Hence it must be $\emptyset \in \mathscr{R}$. But as there are AFs F with $\emptyset \in grd^*(F)$ it must be that $\emptyset \notin \mathscr{R}$. Hence there is no such translation. □

When trying to translate *grd** to *adm* or *com* the only problem comes from handling the empty extension. Admissible semantics always return the empty set as an extension and usually we want to exclude it from the *grd** extensions, except when the empty set is the only *grd** extension. We cannot handle this with the set $\mathscr{R}$. However, one can imagine to enrich the notion of translations such that we can deal with this or simple exclude AFs with the empty set as resolution-based grounded extension. Then by our results on signatures and the results in [8] we know that there are translations with $\mathscr{R} = \{\emptyset\}$, but by the complexity results from Section 3 no efficient translation is possible.

Proposition 11. *Even if we only consider* AF*s F with $grd^*(F) \neq \{\emptyset\}$ there is no efficient translation from grd* to σ for $\sigma \in \{adm, com\}$, unless L = P.*

Proof. Assume that there exists such a translation Tr. Then for an AF $F = (A,R)$ and set $E \subseteq A$ it holds that $E \in grd(F)$ iff $E \in \sigma(Tr(F))$. Thus Tr would be an L-reduction from the P-hard problem Ver_{grd^*} to Ver_σ which is in L [11]. This implies L = P. □

5. Strong Equivalence

This line of research was initiated by Oikarinen and Woltran [9] who introduced the notion of *strong equivalence* for AFs as follows. Two AFs F and G are strongly equivalent under a semantics σ (in symbols $F \equiv_s^\sigma G$) if for any AF H, $\sigma(F \cup H) = \sigma(G \cup H)$. By definition, $F \equiv_s^\sigma G$ implies standard equivalence, i.e. $\sigma(F) = \sigma(G)$; but the other direction is not true in general. A number of further equivalence notions in between strong and standard equivalence have been investigated [10], we consider *normal expansion equivalence* here. An AF H is a *normal expansion*[6] of AF F, if for all $(a,b) \in R_H$ it holds $a \notin A_F$ or $b \notin A_F$. Thus no new attacks between the arguments of F can be introduced in H. Two AFs F and G are normal expansion equivalent under a semantics σ (in symbols $F \equiv_n^\sigma G$) if $\sigma(F \cup H) = \sigma(G \cup H)$ for any AF H being a normal expansion of F and G. Baumann [10] has shown strong equivalence to coincide with normal expansion equivalence under all standard semantics.

A valuable tool for deciding strong equivalence is the notion of a *kernel* of an AF. Informally speaking, kernels are frameworks without redundant attacks. For an AF $F = (A,R)$ and $\sigma \in \{stb, pref, grd\}$, the σ-kernel $F^{k(\sigma)}$ is given by $(A, R^{k(\sigma)})$, whereby

- $R^{k(stb)} = R \setminus \{(a,b) \mid a \neq b, (a,a) \in R\}$,
- $R^{k(pref)} = R \setminus \{(a,b) \mid a \neq b, (a,a) \in R, \{(b,a),(b,b)\} \cap R \neq \emptyset\}$,
- $R^{k(grd)} = R \setminus \{(a,b) \mid a \neq b, (b,b) \in R, \{(a,a),(b,a)\} \cap R \neq \emptyset\}$.

Following, [9,10] it holds that $F \equiv_s^\sigma G$ iff $F \equiv_n^\sigma G$ iff $F^{k(\sigma)} = G^{k(\sigma)}$, for $\sigma \in \{stb, pref, grd\}$. The *pref*-kernel also can be used to decide strong equivalence (resp. normal expansion equivalence) for further semantics, while the *grd*-kernel so far only worked for the grounded semantics. In this section, we show that strong and normal expansion equivalence under the resolution-based grounded semantics can be decided via the *grd*-kernel, as well. Before giving the main result, we need a few lemmas.

Lemma 3. *Let $F = (A,R)$ be an AF and F' be obtained from F by replacing an attack $(b,a) \in R$ where $(b,b) \in R$ by attack (a,b). Then, $grd(F) \subseteq grd(F')$.*

Proof. Consider any S conflict-free in F. Since b is self-attacking in both F and F', we have $b \notin S$, and S is conflict-free in F', regardless of whether $a \in S$ or not. Consider any $c \in \mathscr{F}_F(S)$, i.e., c that is defended by S in F. If $c = a$, then $a \in \mathscr{F}_{F'}(S)$ as $\{d \in A \mid (d,a) \in R_{F'}\} = \{b\} \cup \{d \in A \mid (d,a) \in R\}$. Finally, assume $c \neq a,b$ ($(b,b) \in F$ implies $b \notin S$). Now $c \in \mathscr{F}_{F'}(S)$, as the only difference between F and F' is in the attacks of a and b. Thus $\mathscr{F}_F(S) \subseteq \mathscr{F}_{F'}(S)$ for all conflict-free S and $grd(F) \subseteq grd(F')$ then follows. □

Lemma 4. *For any AF F, $grd^*(F) = grd^*(F^{k(grd)})$.*

Proof. First recall that $A_F = A_{F^{k(grd)}}$ and $R_F \supseteq R_{F^{k(grd)}}$. Thus, if $G \in \gamma(F^{k(grd)})$ then, $G \in \gamma(F)$, i.e. $\gamma(F^{k(grd)}) \subseteq \gamma(F)$. Below we use $\gamma^-(F)$ as shorthand for $\gamma(F) \setminus \gamma(F^{k(grd)})$.

Now, by definition $grd^*(F) = \min\{grd(G) \mid G \in \gamma(F)\} = \min\{grd(G) \mid G \in \gamma(F^{k(grd)}) \cup \gamma^-(F)\}$. We next show that for each $G \in \gamma^-(F)$ there exists a $G' \in \gamma(F^{k(grd)})$, such that $grd(G') \subseteq grd(G)$. Let $G \in \gamma^-(F)$ and Δ be all attacks (a,b) in R_G such that $(a,a) \notin R_F$, $(b,b) \in R_F$ and $(b,a) \in R_F$ (i.e. Δ are the redundant attacks wrt. the grounded

[6] We use a definition slightly different from [10]. The resulting notion of normal expansion equivalence is equivalent to that used in [10].

kernel of F). Note that for all $(a,b) \in \Delta$, $(b,a) \in R_{F^{k(grd)}}$; likewise, all other attacks $R_G \setminus \Delta$ are contained in $R_{F^{k(grd)}}$ as well. Thus there exists an $G' \in \gamma(F^{k(grd)})$, obtained from G by replacing all $(a,b) \in \Delta$ by (b,a). It remains to show that $grd(G') \subseteq grd(G)$. This can be done by sequentially applying Lemma 3. This shows that $\min\{grd(G) \mid G \in \gamma(F^{k(grd)}) \cup \gamma^-(F)\} = \min\{grd(G) \mid G \in \gamma(F^{k(grd)})\} = grd^*(F^{k(grd)})$. $\square$

Proposition 12. *For any* AF*s F and G: $F^{k(grd)} = G^{k(grd)}$ iff $F \equiv_s^{grd^*} G$ iff $F \equiv_n^{grd^*} G$.*

Proof. Suppose $F^{k(grd)} = G^{k(grd)}$ and let H,S s.t. $S \in grd^*(F \cup H)$. We show $S \in grd^*(G \cup H)$. By Lemma 4, $S \in grd^*((F \cup H)^{k(grd)})$. In [9], it was shown (Lemma 7) that if $F^{k(grd)} = G^{k(grd)}$, then $(F \cup H)^{k(grd)} = (G \cup H)^{k(grd)}$ for any AF H. Thus also $S \in grd^*((G \cup H)^{k(grd)})$ and $S \in grd^*(G \cup H)$, again by Lemma 4. By symmetry and definition of strong equivalence, we get $F^{k(grd)} = G^{k(grd)}$ implies $F \equiv_s^{grd^*} G$. Furthermore, this implies $F \equiv_n^{grd^*} G$ (as the set of normal expansions is a subset of all expansions).

We are left to show that $F \equiv_n^{grd^*} G$ implies $F^{k(grd)} = G^{k(grd)}$. Let us assume $F^{k(grd)} \neq G^{k(grd)}$. If $grd^*(F^{k(grd)}) \neq grd^*(G^{k(grd)})$, then $grd^*(F) \neq grd^*(G)$ by Lemma 4, which implies $F \not\equiv_n^{grd^*} G$. Thus, in the following we assume $grd^*(F) = grd^*(G)$.

First consider the case $A_{F^{k(grd)}} \neq A_{G^{k(grd)}}$. By definition this holds iff $A_F \neq A_G$. Wlog. let $a \in A_F \setminus A_G$. Since $a \notin A_G$, we have $a \notin S$ for $S \in grd^*(G) = grd^*(F)$. Let $H = (\{a\}, \emptyset)$. Clearly, $F \cup H = F$ and thus $grd^*(F \cup H) = grd^*(F)$. On the other hand, $a \in S'$ for each $S' \in grd^*(G \cup H)$, since there is no attack on a in $G \cup H$. Consequently, $F \not\equiv_n^{grd^*} G$ since H is a normal expansion.

Now suppose $A_{F^{k(grd)}} = A_{G^{k(grd)}}$, i.e. $A_F = A_G$. Thus wlog. there exists some $(a,b) \in R_{F^{k(grd)}} \setminus R_{G^{k(grd)}}$. Let $c \in \mathfrak{A}$ be a new argument not contained in A_F and $B = A_F \setminus \{a,b\}$. If $a = b$, i.e., $(a,a) \in F$ and $(a,a) \notin G$, we consider a normal expansion $H = \big(B \cup \{c\}, \{(c,d) \mid d \in B\}\big)$. Then, $\{c\} \in grd^*(F \cup H)$ (c is defended by $\emptyset$ in all resolutions of $F \cup H$; no other argument is defended by $\{c\}$ in any resolution of $F \cup H$) and $\{c\} \notin grd^*(G \cup H)$ (c is defended by $\emptyset$ in all resolutions of $G \cup H$ and a is defended by $\{c\}$ in any resolution of $G \cup H$). Hence, we can assume that any self-loop is either contained in both F and G or in none of them.

Finally, consider $a \neq b$. Since $(a,b) \in R_{F^{k(grd)}}$, $(a,b) \in R_F$ and (i) $(b,b) \notin R_F$; or (ii) $(a,a) \notin R_F$ and $(b,a) \notin R_F$.

(i) If $(b,b) \notin R_F$, then $(b,b) \notin R_G$. Moreover, since $(a,b) \notin R_{G^{k(grd)}}$, also $(a,b) \notin R_G$. Let $c \in \mathfrak{A}$ be a new argument not contained in A_F and $B = A_F \setminus \{a,b\}$. We consider a normal expansion $H = \big(A_F \cup \{c\}, \{(c,d) \mid d \in B\}\big)$. In case $(a,a) \in R_F$ and $(a,a) \in R_G$, we have $\bigcup_{F' \in \gamma(F \cup H)} grd(F') = \{\{c\},\{b,c\}\}$ (if $(b,a) \in R_F$) or $\bigcup_{F' \in \gamma(F \cup H)} grd(F') = \{\{c\}\}$ (if $(b,a) \notin R_F$) and thus $grd^*(F \cup H) = \{\{c\}\}$. On the other hand, $\bigcup_{G' \in \gamma(G \cup H)} grd(G') = \{\{b,c\}\}$ (regardless whether $(b,a) \in R_G$ holds) and thus $\{c\} \notin grd^*(G \cup H)$. In case $(a,a) \notin R_F$ and $(a,a) \notin R_G$, we have $grd^*(F \cup H) = \{\{a,c\},\{b,c\}\}$ (if $(b,a) \in R_F$) or $grd^*(F \cup H) = \{\{a,c\}\}$ (if $(b,a) \notin R_F$); whereas $grd^*(G \cup H) = \{\{b,c\}\}$ (if $(b,a) \in R_G$) or $grd^*(G \cup H) = \{\{a,b,c\}\}$ (if $(b,a) \notin R_G$). Thus $grd^*(F \cup H) \neq grd^*(G \cup H)$ follows.

(ii) If $(b,b) \in R_F$, then $(b,b) \in R_G$, $(a,a) \notin R_F$, $(a,a) \notin R_G$, and $(b,a) \notin R_F$. We consider a normal expansion $H = \big(A_F \cup \{c,e\}, \{(c,d) \mid d \in B\} \cup \{(b,e)\}\big)$, where $c,e \in \mathfrak{A}$ are new arguments not contained in A_F. Now, $grd^*(F \cup H) = \{\{a,c,e\}\}$ while $grd^*(G \cup H) = \{\{a,c\}\}$ (if $(b,a) \notin G$) or $grd^*(G \cup H) = \{\{c\}\}$ (if $(b,a) \in G$). Thus $F \not\equiv_n^{grd} G$. $\square$

6. Conclusion

In this paper we have further investigated the resolution-based grounded semantics (*grd**), a multiple-status semantics which is inherited from a particular schema via the (unique status) grounded semantics. In fact, our results revealed this origin: first, we have shown that the verification problem for *grd** is P-complete (which also holds for the grounded semantics). This P-hardness result makes an efficient translation from grounded to *grd** semantics (in contrast to stable or admissible) semantics possible. Second, we have shown that strong equivalence is equally characterized for grounded and *grd** semantics via the *grd*-kernel. An interesting implication of this result is as follows. Since the number of symmetric arguments somehow determines the "search space" for the resolution-based grounded semantics, the notion of a kernel provides a computationally cheap pre-processing technique by reducing the number of symmetric attacks without changing the extensions.

We also have shown that the signature (i.e. all possible sets of extensions a semantics is capable to express) for *grd** semantics looks different compared to other multiple-status semantics. However, *grd** semantics is not "stronger" than preferred semantics. In other words, it holds that for each AF F there exists an AF F' with $grd^*(F) = pref(F')$. This result suggests the question whether *grd** can be efficiently translated to *pref* as natural next step for future work.

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